ON NANO Y¨-CLOSED AND NANO Y¨-OPEN FUNCTIONS

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Abstract: we introduce nano Y-¨ closed functions, nano Y-¨ open functions, nano Y¨∗-closed functions and nano Y¨∗-open functions in nano topological spaces and attain solid classifications of these functions and another new concept of functions called nano Y¨∗-closed functions which are stronger then nano Y-¨ closed functions..

Keywords: nano Y-¨ closed, nano Y-¨ open and nano g-closed.

1 INTRODUCTION

Throughout this paper *NTS* U represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset C of a space *NTS* U, $Ncl(C)$ and $Nint(C)$ denote the nano closure of C and the nano interior of Crespectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. [12] *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let X ⊆ U.*

(1) *The lower approximation of* X *with respect to* R *is the set of all objects, which can be for certain classified as X with respect to* R *and it is denoted by* $L_R(X)$. *That is,* $L_R(X) = \cup_{x\in U}\{R(x) : R(x) \subseteq X\}$, *where* R(x) *denotes the equivalence class determined by x.*

(2) *The upper approximation of* X *with respect to* R *is the set of all objects, which can be possibly classified as X with respect to* R *and it is denoted by* $U_R(X)$. *That is,* $U_R(X) = \cup_{x\in U}\{R(x) : R(x) \cap X \neq \varphi\}$.

(3) *The boundary region of* X *with respect to* R *is the set of all objects, which can be classified neither as* X *nor as not - X with respect to R and it is denoted by* $B_R(X)$. *That is,* $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2. [7] *If (U,R) is an approximation space and X,Y ⊆ U; then*

(1) $L_R(X) \subseteq X \subseteq U_R(X)$;
(2) $L_R(\varphi) = U_R(\varphi) = \varphi$ and $L_R(U) = U_R(U) = U$;
(3) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
(4) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
(5) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
(6) $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
(7) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
(8) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
(9) $U_RU_R(X) = L_RU_R(X) = U_R(X)$;
(10) $L_RL_R(X) = U_RL_R(X) = L_R(X)$;

Definition 2.3. [7] *Let U be the universe,* R *be an equivalence relation on* U *and* $\tau_R(X) = \{U,\varphi,L_R(X),U_R(X),B_R(X)\}$ *where* $X \subseteq U$. *Then by the Property 2.2,* $\tau_R(X)$ *satisfies the following axioms:*

(1) U *and* $\varphi \in \tau_R(X)$,
(2) *The union of the elements of any sub collection of* $\tau_R(X)$ *is in* $\tau_R(X)$,
(3) *The intersection of the elements of any finite subcollection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

That is, $\tau_R(X)$ *is a topology on U called the nano topology on U with respect to X. We call NTS U. The elements of* $\tau_R(X)$ *are called as nano open*

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sets and $[\tau_R(X)]^c$ *is called as the dual nano topology of* $[\tau_R(X)]$.

Remark 2.4. [7] *If* $[\tau_R(X)]$ *is the nano topology on U with respect to* X, *then the set* $B = \{U,\varphi,L_R(X),B_R(X)\}$ *is the basis for* $\tau_R(X)$.

Definition 2.5. [7] *If NTS U with respect to* X *and if* C ⊆ G, *then the nano interior of* C *is defined as the union of all nano open subsets of* C *and it is denoted by* Nint(C). *That is, Nint(C) is the largest nano open subset of* C. *The nano closure of* C *is defined as the intersection of all nano closed sets containing* C *and it is denoted by* Ncl(C).
That is, Ncl(C) *is the smallest nano closed set containing* C.

Definition 2.6. *A subset H of NTS U is called a*
(1) *nano semi-open set* [7] *if* C ⊆ Ncl(Nint(C)).
The complement of nano semi-open set is called nano semi-closed set.
The nano semi-closure [7] *of a subset* C *of* X, *denoted by* Nscl(C) *is defined to be the intersection of all semi-closed sets of NTS U containing* C. *It is known that* Nscl(C) *is a nano semi-closed set.*
(2) *nano generalized closed (briefly nano g-closed) set* [2] *if Ncl(C) ⊆ G whenever C ⊆ G and U is nano open in NTS U.*
The complement of nano g-closed set is called nano g-open set;
(3) *nano semi-generalized closed (briefly nano sg-closed) set* [1] *if Nscl(C) ⊆ G whenever* C ⊆ G *and G is semi-open in NTS U.*
The complement of nano sg-closed set is called nano sg-open set;
(4) *nano generalized semi-closed (briefly nano gs-closed) set* [1] *if Nscl(C) ⊆ G whenever* C ⊆ G *and G is nano open in NTS U.*
The complement of nano gs-closed set is called nano gs-open set;
(5) *nano g∗s-closed set* [13] *if Nscl(C) ⊆ G whenever C ⊆ G and G is nano gs-open in NTS U.*
The complement of nano g∗s-closed set is called nano g∗s-open set.
(6) *nano gˆ-closed set* [9] *(=nano ω-closed) if* Ncl(C) ⊆ G *whenever* C ⊆ G *and* G *is nano semi-open in NTS U.*
The complement of nano gˆ-closed set is called nano gˆ-open set;

Definition 2.7. [3] *A NTS U is said to be nano normal space if for any pair of disjoint nano closed sets C and K, there exists disjoint nano open sets M and N such that C ⊆ M and K ⊆ N.*

Definition 2.8. [8] *A function* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *is called nano ωcontinuous if the inverse image of every nano closed set in NTS V is nano ω-closed set in NTS U.*

Definition 2.9. *A function* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *is called:*
(1) *nano g-closed* [4] *if f(P) is nano g-closed in NTS T for every nano closed set P of NTS U.*
(2) *nano sg-closed* [6] *if f(P) is nano sg-closed in NTS T for every nano closed set P of NTS U.*

Nano ϒ¨-closed functions

Definition 3.1. *In a NTS, a function* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *is called a*
(1) *nano gs-closed if f(P) is nano gs-closed in NTS T for every nano closed set P of NTS U.*
(2) *nano g∗s-closed if f(P) is nano g∗s-closed in NTS T for every nano closed set P of NTS U.*
(3) *nano ϒ¨-closed if the image of every nano closed set in NTS U is nano ϒ¨-closed in NTS T.*
(4) *nano strongly ϒ¨-continuous if the inverse image of every nano ϒ¨-open set in NTS T is nano open in NTS U.*
(5) *Let NTS U be a nano topological space. Let x be a point of U and G be a subset of U. Then G is called a nano ϒ¨-neighborhood of x (briefly, nano ϒ¨nbhd of x) in U if there exists a nano ϒ¨-open set P of U such that x ∈ P ⊂G.*

Example 3.2. *Let* $U = \{a,b,c\}$ *with* $U/R = \{\{a\},\{b,c\}\}$ *and* $X = \{b\}$. *Then the nano topology* $\tau_R(X) = \{\varphi,U,\{b\}\}$. *Let* $V = \{a,b,c\}$ *with* $V/R' = \{\{a\},\{b,c\}\}$ *and* $Y = \{a,b\}$. *Then the nano topology* $\tau_{R'}(Y) = \{\varphi,V,\{a,b\}\}$. *Let* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *be the identity function. Then f is a nano ϒ¨-closed function.*

Proposition 3.3. *A function* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *is nano ϒ¨-closed ⇐⇒Nϒ¨-cl(f(C)) ⊆ f(Ncl(C)) for every subset C of NTS U.*

Proof. Suppose that f is nano ϒ-¨ closed and C ⊆ U. Then Ncl(C) is nano closed in U and so f(Ncl(C)) is nano ϒ-¨ closed in $(V,\tau_{R'}(Y))$. We have f(C) ⊆ f(Ncl(C)) and *N*ϒ-¨ cl(f(C)) ⊆ *N*ϒ-¨ cl(f(Ncl(C))) = *f*(*Ncl*(*C*)).

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Conversely, let C be any nano closed set in *NTS* U. Then C = Ncl(C) and so f(C) = $f(Ncl(C)) \supseteq N\Upsilon$-¨ cl(f(C)), by hypothesis. We have f(C) ⊆ $N\Upsilon$-¨ cl(f(C)). Therefore f(C) = $N\Upsilon$-¨ cl(f(C)).
i.e., f(C) is nano ϒ-¨ closed and hence f is nano ϒ-¨ closed.

Proposition 3.4. *Let* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *be a function such that* $N\Upsilon$*¨cl(f(C)) ⊆ f(Ncl(C)) for every subset C ⊆ U. Then the image f(C) of a nano closed set C in NTS U is nano ϒ¨-closed in NTS V.*

Proof. Let C be a nano closed set in *NTS* U. Then by hypothesis $N\Upsilon$-¨ cl(f(C)) ⊆ $f(Ncl(C)) = f(C)$ and so $N\Upsilon$-¨ cl(f(C)) = $f(C)$. Therefore $f(C)$ is nano ϒ-¨ closed in
NTS V.

Theorem 3.5. *A function* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *is nano ϒ¨-closed ⇐⇒for every subset C of NTS V and avery nano open set G containing* $f^{-1}(C)$ *there is a nano ϒ¨-open set P of NTS V such that C ⊆ P and* $f^{-1}(P) \subseteq G$*.*

Proof. Suppose f is nano ϒ-¨ closed. Let C ⊆ V and G be a nano open set of *NTS* U such that $f^{-1}(C) \subseteq G$. Then P = $(f(G^c))^c$ is a nano ϒ-¨ open set containing S such that $f^{-1}(P) \subseteq G$.
Conversely, let F be a nano closed set of *NTS* U. Then $f^{-1}((f(F))^c) \subseteq F^c$ and F^c is nano open. By assumption, there exists a nano ϒ-¨ open set P in *NTS* V such that $(f(F))^{-1} \subseteq P$ and $f^{-1}(P) \subseteq F^c$ and so $F \subseteq (f^{-1}(P))^c$. Hence $P^c \subseteq f(F) \subseteq f((f^{-1}(P))^c) \subseteq P^c$ which implies f(F) = P^c. Since P^c is nano ϒ-¨ closed, f(F) is nano ϒ-¨ closed and therefore f is nano ϒ-¨ closed.

Proposition 3.6. *If* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *is nano gs-irresolute nano ϒ¨closed and C is a nano ϒ¨-closed subset of NTS U, then f(C) is nano ϒ¨-closed in NTS V.*

Proof. Let G be a nano *gs*-open set in *NTS* V such that f(C) ⊆ G. Since f is nano *gs*-irresolute, $f^{-1}(G)$ is a nano *gs*-open set containing C. Hence Ncl(C) ⊆ $f^{-1}(G)$ as C is nano ϒ-¨ closed in *NTS* U. Since f is nano ϒ-¨ closed, $f(Ncl(C))$ is a nano ϒ-¨ closed set contained in the nano *gs*-open set G, =⇒ $Ncl(f(Ncl(C))) \subseteq G$ and hence $Ncl(f(C)) \subseteq G$. Therefore, f(C) is a nano ϒ-¨ closed set in *NTS* V.

Corollary 3.7. *Let* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *be nano ϒ¨-closed and* $g : (V,\tau_{R'}(Y)) \to (W,\tau R''(Z))$ *be nano ϒ¨-closed and nano gs-irresolute, then* $g \circ f : (U,\tau_R(X)) \to (W,\tau R''(Z))$ *is nano ϒ¨-closed.*

Proof. Let C be a nano closed set of *NTS* U. Then by hypothesis f(C) is a nano ϒ-¨ closed set in *NTS* V. Since *g* is both nano ϒ-¨ closed and nano *gs*-irresolute by Proposition 3.6, $g(f(C)) = (g \circ f)(C)$ is nano ϒ-¨ closed in *NTS* W and therefore $g \circ f$is nano ϒ-¨ closed.

Proposition 3.8. *Let* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$, $g : (V,\tau_{R'}(Y)) \to (W,\tau R''(Z))$ *be nano ϒ¨-closed functions and NTS V be a TNϒ¨-space. Then* $g \circ f : (U,\tau_R(X)) \to (W,\tau R''(Z))$ *is nano ϒ¨-closed.*

Proof. Let C be a nano closed set of *NTS* U. Then by assumption f(C) is nano ϒ-¨ closed in *NTS* V. Since *NTS* V is a TNϒ-¨ space, $f(C)$ is nano closed in *NTS* V and again by assumption $g(f(C))$ is nano ϒ-¨ closed in *NTS* W.
i.e., $(g \circ f)(C)$ is nano ϒ-¨ closed in *NTS* W and so $g \circ f$ is nano ϒ-¨ closed.

Proposition 3.9. *If* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *is nano ϒ¨-closed,* $g : (V,\tau_{R'}(Y)) \to (W,\tau R''(Z))$ *is nano ϒ¨-closed (resp. nano g-closed, nano g∗s-closed, nano sgclosed and nano gs-closed) and NTS V is a TNϒ¨-space, then* $g \circ f : (U,\tau_R(X)) \to (W,\tau R''(Z))$ *is nano ϒ¨-closed (resp. nano g-closed, nano g∗s-closed, nano sg-closed and nano gs-closed).*

Proof. Similar to Proposition 3.8.

Proposition 3.10. *Let* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *be a nano closed function and* $g : (V,\tau_{R'}(Y)) \to (W,\tau R''(Z))$ *be a nano ϒ¨-closed function, then their* $g \circ f : (U,\tau_R(X)) \to (W,\tau R''(Z))$ *is nano ϒ¨-closed.*

Proof. Similar to Proposition 3.8.

Theorem 3.11. *Let* $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ *and* $g : (V,\tau_{R'}(Y)) \to (W,\tau R''(Z))$ *be two functions such that their* $g \circ f : (U,\tau_R(X)) \to (W,\tau R''(Z))$ *is a nano ϒ¨-closed function. Then the next conditions are true.*

(1) *If f is nano continuous and surjective, then g is nano ϒ¨-closed.*

(2) *If g is nano ϒ¨-irresolute and injective, then f is nano ϒ¨-closed.*

(3) *If f is nano g^-continuous, surjective and NTS U is a* $TN_{g\hat{}}$*-space, then g is nano ϒ¨-closed.*

(4) *If g is strongly nano ϒ¨-continuous and injective, then f is nano closed.*

Proof. (1) Let C be a nano closed set of *NTS* V. Since f is nano continuous, $f^{-1}(C)$ is nano closed in *NTS* U and since $g \circ f$ is nano ϒ-¨ closed, $(g \circ f)(f^{-1}(C))$ is nano ϒ-¨ closed in *NTS* W. That is g(C) is nano ϒ-¨ closed in *NTS* W, since f is

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surjective. Therefore g is a nano ϒ-̈ closed function.

(2) Let C be a nano closed set of *NTS* U. Since g ∘ f is nano ϒ-̈ closed, (g ∘ f) (C) is nano ϒ-̈ closed in *NTS* W. Since g is nano ϒ-̈ irresolute, $g^{-1}((g \circ f)(C))$ is nano ϒ-̈ closed set in *NTS* V. That is f(C) is nano ϒ-̈ closed in *NTS* V, since *g* is injective. Thus f is a nano ϒ-̈ closed function.

(3) Let C be a nano closed set of *NTS* V. Since f is nano $\hat{g}$-continuous, $f^{-1}(C)$ is nano $\hat{g}$-closed in *NTS* U. Since *NTS* U is a $TN_{\hat{g}}$-space, $f^{-1}(C)$ is nano closed in *NTS* U and so as in (1), *g* is a nano ϒ-̈ closed function.

(4) Let C be a nano closed set of *NTS* U. Since g ∘ f is nano ϒ-̈ closed, (g ∘ f)(C) is nano ϒ-̈ closed in *NTS* W. Since g is nano strongly ϒ-̈ continuous, $g^{-1}((g \circ f)(C))$ is nano closed in *NTS* V. That is f(C) is nano closed set in *NTS* V, since g is injective. Therefore f is a nano closed function.

Theorem 3.12. *If f : $(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is a nano continuous, nano ϔ-closed function from a nano normal space NTS U onto a space NTS V, then NTS V is nano normal.*

Proof. Let C and K be two disjoint nano closed subsets of *NTS* V. Since *f* is nano continuous, $f^{-1}(C)$ and $f^{-1}(K)$ are distinct nano closed sets of *NTS* U. Since *NTS* U is nano normal, there exist disjoint nano open sets G and P of *NTS* U such that $f^{-1}(C) \subseteq G$ and $f^{-1}(K) \subseteq P$. Since f is nano ϒ-̈ closed, by Theorem 3.5, there exist distinct nano ϒ-̈ open sets G_1 and C_1 in *NTS* V such that $C \subseteq G_1$, $K \subseteq C_1$, $f^{-1}(G_1) \subseteq G$ and $f^{-1}(C_1) \subseteq P$. Since G and P are disjoint, int(G_1) and int(C_1) are distinct nano open sets in *NTS* V. Since C is nano closed, C is nano *gs*-closed and therefore $C \subseteq$ Nint(G_1). Similarly $K \subseteq$ Nint(C_1) and hence *NTS* V is nano normal.

Definition 3.13. *A function f : $(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is said to be a nano ϔopen function if the image f(C) is nano ϔ-open in NTS V for each nano open set C in NTS U.*

Proposition 3.14. *For any bijection f : $(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$, the next conditions are equivalent:*

(1) *$f^{-1} : (V,\tau_{R'}(Y)) \to (U,\tau_R(X))$ is nano ϔ-continuous.*

(2) *f is nano ϔ-open function.*

(3) *f is nano ϔ-closed function.*

Proof. (1) ⇒ (2). Let G be a nano open set of *NTS* U. By assumption, $(f^{-1})^{-1}(G)$ = f(G) is nano ϒ-̈ open in *NTS* V and so f is nano ϒ-̈ open.

(2) ⇒ (3). Let F be a nano closed set of *NTS* U. Then F^c is nano open set in *NTS* U. By assumption, $f(F^c)$ is nano ϒ-̈ open in *NTS* V. That is $f(F^{-1}) = (f(F))^{-1}$ is nano ϒ-̈ open in *NTS* V and therefore *f*(*F*) is nano ϒ-̈ closed in *NTS* V. Hence *f* is nano ϒ-̈ closed.

(3) ⇒ (1). Let F be a nano closed set of *NTS* U. By assumption, *f*(*F*) is nano ϒ-̈ closed in *NTS* V. But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is nano ϒ-̈ continuous.

Theorem 3.15. *Let f :$(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be a function. Then the next conditions are equivalent:*

(1) *f is a nano ϔ-open function.*

(2) *For a subset C of NTS U, f(Nint(C)) ⊆ Nϔ-int(f(C)).*

(3) *For each x ∈ U and for each neighborhood G of x in NTS U, there exists a nano ϔ-neighborhood W of f(x) in NTS V such that W ⊂ f(G).*

Proof. (1) ⇒ (2). Suppose *f* is nano ϒ-̈ open. Let C ⊆ U. Then *Nint*(*C*) is nano open in *NTS* U and so *f*(*Nint*(*C*)) is nano ϒ-̈ open in *NTS* V. We have *f*(*Nint*(*C*)) ⊆ *f*(*C*). Therefore *f*(*Nint*(*C*)) ⊆ *N*ϒ-̈ *int*(*f*(*C*)).

(2) ⇒ (3). Suppose (2) holds. Let x ∈ U and G be an arbitrary neighborhood of x in *NTS* U. Then there exists a nano open set G_1 such that $x \in G_1 \subseteq G$. By assumption, f(G_1) = f(Nint(G_1)) ⊆ *N*ϒ-̈ int(f(G_1)). This implies f(G_1) = *N*ϒ-̈ int(f(G_1)). We have f(G_1) is nano ϒ-̈ open in *NTS* V. Further, f(x) ∈ f(G_1) ⊆ f(G) and so (3) holds, by taking W = f(G_1).

(3) ⇒ (1). Suppose (3) holds. Let G be any nano open set in *NTS* U, x ∈ G and f(x) = y. Then y ∈ f(G) and for each y ∈ f(G), by assumption there exists a nano ϒ-̈ neighborhood W_y of y in *NTS* V such that $W_y \subseteq$ f(G). Since W_y is a nano ϒ-̈ neighborhood of y, there exists a nano ϒ-̈ open set P_y in *NTS* V such that $y \in P_y \subseteq W_y$. Therefore, f(G) = ∪{P_y: *y* ∈ *f*(*G*)} is a nano ϒ-̈ open set in *NTS* V. Thus f is a nano ϒ-̈ open function.

Theorem 3.16. *A function f : $(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano ϔ-open ⇐⇒for any subset C of NTS V and for any nano closed set F containing*

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$f^{-1}(C)$, there exists a nano Ϋ-closed set K of NTS V containing S such that $f^{-1}(K) \subseteq F$.

Proof. Similar to Theorem 3.5.

Corollary 3.17. *A function $f : (U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is nano Ϋ-open $\Leftarrow\Rightarrow f^{-1}(N$Ϋ$\text{-}cl(K)) \subseteq Ncl(f^{-1}(K))$ for each subset K of NTS V.*

Proof. Suppose that *f* is nano Ϋ-open. Then for any K ⊆ V, $f^{-1}(K) \subseteq Ncl(f^{-1}(K))$. By Theorem 3.16, there exists a nano Ϋ-closed set K of *NTS* V such that K ⊆ K_1 and $f^{-1}(K_1) \subseteq Ncl(f^{-1}(K))$. Therefore, $f^{-1}(N$Ϋ-cl(B)) ⊆ $(f^{-1}(K_1)) \subseteq Ncl(f^{-1}(K))$, since K_1 is a nano Ϋ-closed set in *NTS* V.

Conversely, let C be any subset of *NTS* V and F be any nano closed set containing $f^{-1}(C)$. Put K_1 = *N*Ϋ-cl(C). Then K_1 is a nano Ϋ-closed set and C ⊆ K_1.

By assumption, $f^{-1}(K_1) = f^{-1}(N$Ϋ-cl(C)) ⊆ $Ncl(f^{-1}(C))$ ⊆ F and therefore by Theorem 3.16, f is nano Ϋ-open.

Nano Ϋ∗-closed functions

Definition 4.1. *A function $f : (U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is said to be nano Ϋ∗-closed if the image f(C) is nano Ϋ-closed in NTS V for every nano Ϋ-closed set C in NTS U.*

Proposition 4.2. *A function $f : (U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is nano Ϋ∗-open $\Leftarrow\Rightarrow N$Ϋ-cl(f(C)) ⊆ f(NΫ-cl(C)) for every subset C of NTS U.*

Proof. Similar to Proposition 3.3.

Proposition 4.3. *For any bijection $f : (U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$, the next conditions are equivalent:*

(1) *$f^{-1} : (V,\tau_{R'}(Y)) \rightarrow (U,\tau_R(X))$ is nano Ϋ-irresolute.*

(2) *f is nano Ϋ∗-open function.*

(3) *f is nano Ϋ∗-closed function.*

Proof. Similar to Proposition 3.14.

Proposition 4.4. *If $f : (U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is nano gs-irresolute and nano Ϋ-closed, then it is a nano Ϋ∗-closed function.*

Proof. The Proof follows from Proposition 3.6.

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