GINI SIMPSON INDEX FOR PICTURE FUZZY SETS WITH THEIR APPLICATION IN MULTI -ATTRIBUTE DECISION MAKING PROCESS

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Abstract: In present paper, we proposed a Gini Simpson index for picture fuzzy set with their application in MADM and discuss it's properties which are investigated in a mathematical framework. We developed an algorithm based on TODIM(An acronym in Portuguess for interactive multi-attribute decision making) which we applied ton the proposed entropy to solve the MADM problems under the picture fuzzy environment when the criteria weights are completely known. With took a numerical example on Muthoot Finance Limited to demonstrate the applicability and feasibility of the proposed approach.

Keyword: Picture fuzzy set(PFS) , Hamming distance, Picture fuzzy number (PFN), TODIM
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I. Introduction

Atanassov [2] extended the idea of FS given by Zadeh [1] to intuitionistic fuzzy sets (IFSs) .The application of IFSs have investigated by many authors. A characterization of IFS namely (PFS) developed by Cuong [3] with positive (υ), neutral (ν) , negative (η) and refusal membership degree/ garde (ϕ), respectively. To measure the resemblence between PFS Wei [4] suugested various procedures. An algorithm for PFS based on new distance measure was proposed by Peng and Dai [5] for the decision making process. Son [6] introduced some clustering algorithms while describing the benefities and reasons of using PFSs. Many previous studies have been used in the MADM problems with PF information [4, 7, 22,28,29,33]. Gomes and Lima [8] was the first who introduced the TODIM model for the decision making problems which consists of hesitation and risk. Many scholars have suggested some TODIM approaches [9-15]. Recently, PFS is a method to deal uncertainty in real-world situations. From probabilistic point of view researchers has not done too much work for picture fuzzy numbers (PFNs) with entropy and TODIM method. To overcome this limitatation many researchers extend TODIM method to MADM with PFNs [17, 18,21,].

The main objective of the this paper is to develop a new PF information measure and this information measure have been tested in MADM problems with TODIM approach . To check the feasibility of the proposed approach we made a practical example and compare the results with other existing methods.

The classification of this paper is as follows.In first section we discussed the work done by some researchers in this field . In section 2, we review some indispensable concepts,definitions and a new PF information measure and validated . In section 3,we used the practical example to validate the proposed picture fuzzy entropy measure. Finally, the concluding remarks and future scope of this study are discussed in the last section .

2 Preliminaries (concepts and methods)

Some basic definitions and concepts of IFS and PFS are discuss in this section.

Definition 2.1 . *An IFS* G^* *in* J *is defined by [2] as:*

$$G^* = \{(p_i, \upsilon_G(p_i), \eta_G(p_i)) : p_i \in J\}, \quad (2.1)$$

where

$$\upsilon_{G^*} : J \to [0,1] \text{ and } \eta_G : J \to [0,1],$$

with $0 \le \upsilon_{G^*}(p_i) + \eta_{G^*}(p_i) \le 1,$ for all $p_i \in J \ (1 \le i \le n).$ The numbers $\upsilon_{G^*}(p_i)$ and $\eta_{G^*}(p_i)$, repectively, denote the membership and non membership degree of G^*. For an IFS, the pair $(\upsilon_{G^*}(p_i), \nu_{G^*}(p_i))$ is called intuitionistic fuzzy number (IFN).

For each IFS G^* in J, the number $\phi_{G^*}(p_i) = 1 - \upsilon_{G^*}(p_i) - \eta_{G^*}(p_i), p_i \in J$ represents hesitancy degree of p_i in J. Obviously, when $\phi_{G^*}(p_i) = 0,$ that is $\eta_{G^*}(p_i) = 1 - \upsilon_{G^*}(p_i)$ for all $p_i \in J,$ IFS G^* alters an ordinary FS.

Definition 2.2 *A PFS* G^* *on set* J *is defined by [3] as:*

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$G^* = \{(p_i, \upsilon_{G^*}(p_i), \nu_{G^*}(p_i), \eta_{G^*}(p_i)) : p_i \in J\}$ (2.2)

where

$\upsilon_{G^*}: J \to [0,1], \nu_{G^*}: J \to [0,1], \eta_{G^*}: J \to [0,1]$,

and $\upsilon_{G^*}(p_i), \nu_{G^*}(p_i), \eta_{G^*}(p_i) \in [0,1]$, repectively, denote the positive, neutral and negative membership degrees of set G with the condition $0 \leq \upsilon_{G^*}(p_i) + \nu_{G^*}(p_i) + \nu_{G^*}(p_i) \leq 1$, for all $p_i \in J$. Moreover, a degree of refusal membership $\phi_{G^*}(p_i)$ of p_i in G^* can be estimated accordingly as:

$\phi_{G^*}(p_i) = 1 - \upsilon_{G^*}(p_i) - \nu_{G^*}(p_i) - \eta_{G^*}(p_i)$ (2.3)

When $\nu_{G^*}(p_i) = 0$, then the PFSs reduce into IFS, while if $\nu_{G^*}(p_i), \eta_{G^*}(p_i) = 0$ then the PFS becomes FS.

For convenience, the pair $G = (\upsilon_{G^*}(p_i), \nu_{G^*}(p_i), \eta_{G^*}(p_i), \phi_{G^*}(p_i))$ is called a PFN and every PFN represented by $\beta = (\upsilon_\beta, \nu_\beta, \eta_\beta, \phi_\beta)$, where $\upsilon_\beta \in [0,1], \nu_\beta \in [0,1], \eta_\beta \in [0,1], \nu_\beta \in [0,1], \phi_\beta \in [0,1]$ and $\upsilon_\beta + \nu_\beta + \eta_\beta + \phi_\beta = 1$. Sometimes, we omit ϕ_β and in short, we denote a PFN as $\beta = (\upsilon_\beta, \nu_\beta, \eta_\beta)$.

Definition 2.3 *[3, 6] The hamming distance measure between two PFN* $\beta_1 = (\upsilon_{\beta_1}, \nu_{\beta_1}, \eta_{\beta_1})$ *and* $\beta_2 = (\upsilon_{\beta_2}, \nu_{\beta_2}, \eta_{\beta_2})$ *is computed as follows:*

$d_H(\beta_1, \beta_2) = \frac{1}{3}[(|\upsilon_{\beta_1} - \upsilon_{\beta_2}|) + (|\nu_{\beta_1} - \nu_{\beta_2}|) + (|\eta_{\beta_1} - \eta_{\beta_2}|)]$ (2.4)

Definition 2.4 *For every two PFSs* G^* *and* H^*, *Cuong et al.[3, 17] defined some operations in the universe* J *as following.*

1. $G^* \subseteq H^*$ iff $\forall\ p_i \in J$, $\upsilon_{G^*}(p_i) \leq \upsilon_{H^*}(p_i), \nu_{G^*}(p_i) \leq \nu_{H^*}(p_i), \eta_{G^*}(p_i) \geq \eta_{H^*}(p_i)$;
2. $G^* = H^*$ iff $\forall\ p_i \in J, G^* \subseteq H^*$ and $H^* \subseteq G^*$;
3. $G^* \cap H^* = \{\upsilon_{G^*}(p_i) \wedge \upsilon_{H^*}(p_i)\ ,\ \nu_{G^*}(p_i) \wedge \upsilon_{H^*}(p_i)$,and $\eta_{G^*}(p_i) \vee \eta_{H^*}(p_i) | p_i \in J\}$;
4. $G^* \cup H^* = \{\upsilon_{G^*}(p_i) \vee \upsilon_{H^*}(p_i),\ \nu_{G^*}(p_i) \wedge \upsilon_{H^*}(p_i)$, and $\eta_{G^*}(p_i) \wedge \eta_{H^*}(p_i) | p_i \in J\}$.
5. If $G^* \subseteq H^*$ and $H^* \subseteq P$ then $G^* \subseteq P$;
6. $(G^{*c})^c = G^*$;
7. coG^* = $G^{*c} = \{(p_i, \eta_{G^*}(p_i) \nu_{G^*}(p_i), \upsilon_{G^*}(p_i) | p_i \in J)\}$.

We inducted the following comparison law to compare the two PFNs.

Definition 2.5 *[19] Let* $\beta_1 = (\upsilon_{\beta_1}, \nu_{\beta_1}, \eta_{\beta_1})$ *and* $\beta_2 = (\upsilon_{\beta_2}, \nu_{\beta_2}, \eta_{\beta_2})$ *be two PFNs.and its the score function values are denoted by score*(β_1, β_2) *also* $H(\beta_i)(i = 1,2)$ *be the accuracy degree then:*

- If $score(\beta_1) < score(\beta_2)$,then $\beta_1 < \beta_2$;
- If $score(\beta_1) = score(\beta_2)$,then $H(\beta_1) < H(\beta_2)$, shows that β_1 is inferior to β_2, denoted by $\beta_1 < \beta_2$.
  If $H(\beta_1) = H(\beta_2)$, shows that β_1 and β_2, are equivalent and denoted by $\beta_1 \equiv \beta_2$;

Definition 2.6 *Wang et al. [19] introduced some laws for any PFNs* $\beta_1 = (\upsilon_{\beta_1}, \nu_{\beta_1}, \eta_{\beta_1})$, $\beta_2 = (\upsilon_{\beta_2}, \nu_{\beta_2}, \eta_{\beta_2})$.

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(1).
$\beta_1 \otimes \beta_2 = (\upsilon_{\beta_1} + \nu_{\beta_1})(\upsilon_{\beta_2} + \nu_{\beta_2}) - \nu_{\beta_1}\nu_{\beta_2}, \nu_{\beta_1}\nu_{\beta_2}, 1 - (1 - \eta_{\beta_1})(1 - \eta_{\beta_2});$

(2).
$\beta_1^n = (\upsilon_{\beta_1} + \nu_{\beta_1}) - \nu_{\beta_1}^n, \nu_{\beta_1}^n\ 1 - (1 - \eta_{\beta_1})^n$ for $n > 0.$

2.1 Fuzzy Entropy for PFSs

In FS- theory, the fuzzy entropy measure the uncertainty and shows the degree of fuzziness of a FS. To measure the fuzziness Luca and Term [20] introduced the following set of axioms :

Definition 2.7 *A real function* $\widetilde{E^*} \to [0, \infty)$ *is called fuzzy entropy if it fulfills the following chracteristics:*

A1 (Sharpness): For all $G \in FS(J), \widetilde{E^*}(G) = 0$ is minimum iff G is crisp set, i.e.,$\mu_G = 0.5$ for all $G \in FS(J)$.

A2 (Maximality): The value of $\widetilde{E^*}(G)$ is maximum iff G is the most fuzzy set.

A3 (Resolution):$\widetilde{E^*}(G) \geq \widetilde{E^*}(G^*)$, where G^* is the sharpened version .

A4 (Symmetry): $\widetilde{E^*}(G) = \widetilde{E^*}(G^c)$, where $\hat{E}(G^c)$ is the complement set of G. Since for an IFS, $\upsilon + \nu + \eta = 1$, therfore, considering (υ, ν, η) as probability distribution .

Definition 2.8 By Hung and Yang *[16] A real valued function* $\Theta: IFS(J) \to [0, \infty)$ *is called an entropy on* IFS *if it fulfills the below said characteristics:*

(i) Sharpness: $\Theta(G) = 0 \Leftrightarrow \Theta$ is a crisp set.

(ii) Maximality: $\Theta(G) = 1$, maximum value will be attained $\Leftrightarrow \upsilon_G(p_i) = \nu_G(p_i) = \phi_G(p_i) = \frac{1}{3}$, for all $p_i \in J$

(iii) Symmetry: $\Theta(G) = \Theta(G^c)$.

(iv) Resolution: $\Theta(G) \leq \Theta(H) \Leftrightarrow G \subseteq H$,i.e., $\upsilon_G \leq \upsilon_H$ and $\nu_G \leq \nu_H$ for max $(\upsilon_H, \nu_H) \leq \frac{1}{3}$ and $\upsilon_G \geq \upsilon_H$ and $\nu_G \geq \nu_H$ for min $(\upsilon_H, \nu_G) \geq \frac{1}{3}$.

The four components which describes the parametric characterization in PFSs are represented by $(\upsilon, \nu, \eta, \phi)$ also satisfies the conditions $0 \leq \upsilon, \nu, \eta, \phi \leq 1$ and $\upsilon + \nu + \eta + \phi = 1.$

Keeping these concepts in mind, Hung and Yang [16] demonstrate a definition of entropy for PFSs as :

Definition 2.9 *A real function* $ent: PFSs(J) \to [0, \infty)$ *is an entropy on* PFS *if* Ent *holds the four conditions given below:*

(H1) Sharpness: $ent(G) = 0 \Leftrightarrow G$ is a crisp set.

(H2) Maximality: $ent(G) = 1$, The maximum value will be attained $\Leftrightarrow$ $\upsilon_{ent}(p_i) = \nu_{ent}(p_i) = \eta_{ent}(p_i) = \phi_{ent}(p_i) = \frac{1}{4}$, for all $p_i \in J$.

(H3) Symmetry: $ent(G) = ent(G^c)$.

(H4) Resolution: $ent(G) \leq ent(H)$ if G is less fuzzy than F, that is $\upsilon_G \leq \upsilon_H, \nu_G \leq \nu_H$ and $\eta_G \leq \eta_H$ for max $(\upsilon_H, \nu_H, \eta_H) \leq \frac{1}{4}$ and $\upsilon_G \geq \upsilon_H, \nu_G \geq \nu_H$ and $\eta_G \geq \eta_H$ for min $(\upsilon_H, \nu_H, \eta_H) \geq \frac{1}{4}$.

2.2 A New Parametric Measure for PFSs

For any $G \in PFSs,$ we define

$$M_2(G) = \frac{1}{k}\sum_{i=1}^{k} [1 - (\upsilon_G(p_i)^2 + \nu_G(p_i)^2 + \eta_G(p_i)^2 + \phi_G(p_i)^2)]. \quad (2.5)$$

Particular Cases:

1. If $\nu_G(p_i) = 0$ (nuteral membership), then (2.5) reduces to IF Gini Simpson index entropy.

$$i.e., M_2(G) = \frac{1}{k}\sum_{i=1}^{k} [1 - (\upsilon_G(p_i)^2 + \eta_G(p_i)^2 + \phi_G(p_i)^2)]. \quad (2.6)$$

2.If $\nu_G(p_i) = 0$ and $\phi_G(p_i) = 0$ then PF entropy reduces to Fuzzy entropy for Gini Simpson Index:

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$$M_2(G) = \frac{1}{k}\sum_{i=1}^{k} [1 - (\upsilon_G(p_i)^2 + \eta_G(p_i)^2)].$$ (2.7)

2.2.1 Justification

First we will prove the following characteristics before proving the existence of proposed measure .

Property 2.1:

Under the condition $Q4$, we have

$$\left|\upsilon_G(p_i) - \frac{1}{4}\right| + \left|\nu_G(p_i) - \frac{1}{4}\right| + \left|\eta_G(p_i) - \frac{1}{4}\right| + \left|\phi_G(p_i) - \frac{1}{4}\right|$$

$$\geq \left|\upsilon_H(p_i) - \frac{1}{4}\right| + \left|\nu_H(p_i) - \frac{1}{4}\right| + \left|\eta_H(p_i) - \frac{1}{4}\right| + \left|\phi_H(p_i) - \frac{1}{4}\right|$$ (2.8)

and

$$\left[\upsilon_G(p_i) - \frac{1}{4}\right]^2 + \left[\nu_G(p_i) - \frac{1}{4}\right]^2 + \left[\eta_G(p_i) - \frac{1}{4}\right]^2 + \left[\phi_G(p_i) - \frac{1}{4}\right]^2$$

$$\geq \left[\upsilon_H(p_i) - \frac{1}{4}\right]^2 + \left[\nu_H(p_i) - \frac{1}{4}\right]^2 + \left[\eta_H(p_i) - \frac{1}{4}\right]^2 + \left[\phi_H(p_i) - \frac{1}{4}\right]^2$$ (2.9)

Proof: If $\upsilon_G(p_i) \leq \upsilon_H(p_i), \nu_G(p_i) \leq \nu_H(p_i)$ and $\eta_G(p_i) \leq \eta_H(p_i)$ with $\frac{1}{4} \geq$ max $\{\upsilon_H(p_i), \upsilon_H(p_i), \eta_H(p_i)\}$ then $\upsilon_G(p_i) \leq \upsilon_H(p_i) \leq \frac{1}{4}, \nu_G(p_i) \leq \nu_H(p_i) \leq \frac{1}{4}, \eta_G(p_i) \leq \eta_H(p_i) \leq \frac{1}{4}$ and $\phi_G(p_i) \leq \phi_H(p_i) \geq \frac{1}{4}$ which shows that (2.8) and (2.9) hold. Similarly, if $\upsilon_G(p_i) \geq \upsilon_H(p_i), \nu_G(p_i) \geq \nu_H(p_i), \eta_G(p_i) \geq \eta_H(p_i) \leq \frac{1}{4}$ with max $\{\upsilon_H(p_i), \nu_H(p_i), \nu_H(p_i) \geq \frac{1}{4}\}$ then (2.8) and (2.9) hold. Szmidt and Kacpryzk[26] proposed the distance between two IFSs as the distance between their parametres, (υ, μ, η). The Euclidean distance or Hamming distance measure are used to determine the distance between two IFSs.

In PFSs we have four parametres ($\upsilon, \mu, \eta, \phi$), thus, extendig the idea of distance measure from IFSs to PFSs, it may be concluded from property (2.1) PFS H is nearer to the maximum value $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ than PFS G.

Theorem 2.1 *Proposed measure (2.5) is a valid PF measure for* $PFSs$.

Proof. To establish (2.5) as a valid entropy measure for $PFSs$ then we have to show that it saisfies all properties of definition (2.6).

H1: Let G is a crisp set with membership values either $\upsilon_G(p_i) = 1,$ and $\nu_G(p_i) = \eta_G(p_i) = \phi_G(p_i) = 0$ or $\nu_G(p_i) = 1,$ and $\upsilon_G(p_i) = \eta_G(p_i) = \phi_G(p_i) = 0$ or $\eta_G(p_i) = 1$ and $\upsilon_G(p_i) = \nu_G(p_i) = \phi_G(p_i) = 0$ or $\phi_G(p_i) = 1$ and $\upsilon_G(p_i) = \nu_G(p_i) = \eta_G(p_i) = 0.$

$$\Rightarrow 1 - (\upsilon_G(p_i)^2 + \nu_G(p_i)^2 + \eta_G(p_i)^2 + \phi_G(p_i)^2) = 0.$$

Conversely, if $M_2(G) = 0,$ we have

$$1 - (\upsilon_G(p_i)^2 + \nu_G(p_i)^2 + \eta_G(p_i)^2 + \phi_G(p_i)^2) = 0.$$

this is possible only in the following cases:

1. either $\upsilon_G(p_i) = 0$ and $\nu_G(p_i) = \eta_G(p_i) = \phi_G(p_i) = 0$ or
2. $\nu_G(p_i) = 1,$ and $\upsilon_G(p_i) = \eta_G(x_I) = \phi_G(p_i) = 0$ or
3. $\eta_G(p_i) = 0$ and $\upsilon_G(p_i) = \nu_G(x_I) = \phi_G(p_i) = 0$ or
4. $\phi_G(p_i) = 1$ and $\upsilon_G(p_i) = \nu_G(p_i) = \eta_G(p_i) = 0.$

The above results shows that, G is a crisp set iff $M_2(G) = 0$. (H1) is proved.

H2: Since $\upsilon_G(p_i) + \nu_G(p_i) + \eta_G(p_i) + \phi_G(p_i) = 1,$ to find the the maximim value of PF entropy $M_2(G)$, let

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$g(\upsilon_G, \nu_G, \phi_G) = \upsilon_G(p_i) + \nu_G(p_i) + \eta_G + \phi_G(p_i) - 1$

, we form the Lagrange's function as:

$$G^*(\upsilon_G, \nu_G, \phi_G) = M_2(\upsilon_G, \nu_G, \eta_G, \phi_G) + \lambda h(\upsilon_G, \nu_G, \eta_G, \phi_G). \quad (2.10)$$

Where λ is a Lagrange's multipliers.

The maximum value of $M_2(G)$ will be obtained by differentiating (2.10) partially w.r.t. $\upsilon_M, \nu_M, \eta_M, \phi_M$ and λ and equating with zero, we get $\upsilon_G(p_i) = \nu_G(p_i) = \eta_G(p_i) = \phi_G(p_i) = \frac{1}{4}$. The stationary point of $M_2(G)$ is $\upsilon_G(p_i) = \nu_G(p_i) = \eta_G(p_i) = \phi_G(p_i) = \frac{1}{4}$.

Definition 2.10 ***(Hessian)****The Hessian matrix of a function* $\rho(x_1, x_2, x_3, x_4)$ *of four variables is given by*

$$HEN\ (\rho) = \begin{bmatrix} \frac{\partial^2 \rho}{\partial x_1^2} & \frac{\partial^2 \rho}{\partial x_2 \partial x_1} & \frac{\partial^2 \rho}{\partial x_3 \partial x_1} & \frac{\partial^2 \rho}{\partial x_4 \partial x_1} \\ \frac{\partial^2 \rho}{\partial x_1 \partial x_2} & \frac{\partial^2 \rho}{\partial x_2^2} & \frac{\partial^2 \rho}{\partial x_3 \partial x_2} & \frac{\partial^2 \rho}{\partial x_4 \partial x_2} \\ \frac{\partial^2 \rho}{\partial x_1 \partial x_3} & \frac{\partial^2 \rho}{\partial x_2 \partial x_3} & \frac{\partial^2 \rho}{\partial x_3^2} & \frac{\partial^2 \rho}{\partial x_4 \partial x_3} \\ \frac{\partial^2 \rho}{\partial x_1 \partial x_4} & \frac{\partial^2 \rho}{\partial x_2 \partial x_4} & \frac{\partial^2 \rho}{\partial x_3 \partial x_4} & \frac{\partial^2 \rho}{\partial x_4^2} \end{bmatrix}, \quad (2.11)$$

ρ is strictly convex or Concave at a point in its domain if $[HEN](\rho)$ is either positive or negative definite and The Hessian of $M_2(G)$ is given by

$$[\mathrm{HEN}](\mathrm{M_2(G)}) = 2\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (2.12)$$

which is negative definite .Thus, $M_2(G)$ is strictly concave function and attain it's maximum value at $\upsilon_G(p_i) = \nu_G(p_i) = \eta_G(p_i) = \phi_G(p_i) = \frac{1}{4}$.

H3: Since, $M_2(G)$ is a concave function and $E \in PFS(J)$, with maximum value at stationary point, if max $\{\upsilon_G(p_i), \nu_G(p_i), \eta_G(p_i), \phi_G(x_I)\} \le \frac{1}{4}$,then $\upsilon_G(p_i) \le \upsilon_H(p_i), \nu_G(p_i) \le \nu_H(p_i)$ and $\eta_G(p_i) \le \eta_H(p_i)$ implies $\phi_G(p_i) \ge \phi_H(p_i) \ge \frac{1}{4}$. Therefore, we see that $M_2(G)$ satisfies the condition $H4$.

Similarly, if min $\{\upsilon_G(p_i), \nu_G(p_i), \eta_G(p_i)\} \ge \frac{1}{4}$,then $\upsilon_G(p_i) \le \upsilon_H(p_i), \nu_G(p_i) \ge \nu_H(p_i)$ and $\eta_G(p_i) \ge \eta_H(p_i)$. Again ,by using property (2.1),we observe that $M_2(G)$ satisfies condition $H4.$

H4: For any PFS , $M_2(G) = M_2(G^c)$, which is clear from the definition.

Hence the theorem proved.

Theorem 2.2 *For* $G, H \in PFS(J)$*, such that for all* $p_i \in J$ *either* $G \subseteq H$ *or* $H \subseteq G;$ *then,*

$$M_2(G \cup H) + M_2(G \cap H) = M_2(G) + M_2(H) \quad (2.13)$$

Proof.For the proof of this theorem , seperate J into two parts say Z_1, Z_2, such that

$$Z_1 = \{p_i \in J : G \subseteq H\}, \text{and} \qquad Z_2 = \{p_i \in J : G \supseteq H\} \quad (2.14)$$

$$\upsilon_G(p_i) \le \upsilon_H(p_i), \nu_G(p_i) \le \nu_H(p_i), \eta_G(p_i) \ge \eta_H(p_i) \quad \forall \quad p_i \in Z_1 \quad (2.15)$$

$$\upsilon_G(p_i) \ge \upsilon_H(p_i), \nu_G(p_i) \ge \nu_H(p_i), \eta_G(p_i) \ge \eta_H(p_i) \quad \forall \quad p_i \in Z_2 \quad (2.16)$$

$$\text{Now, } M_2(G \cup H) = \frac{1}{n}\sum_{i=1}^{n} [1 - (\upsilon_{(G\cup H)}(p_i)^2 + \nu_{(G\cup H)}(p_i)^2 + \eta_{(G\cup H)}(p_i)^2 + \phi_{(G\cup H)}(p_i)^2)] \quad (2.17)$$

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$$=\frac{1}{n}\sum_{Z_1}\left[\left(1-\left(\upsilon_H(p_i)^2+\nu_H(p_i)^2+\phi_H(p_i)^2\right)\right)\right]$$

$$+\frac{1}{n}\sum_{Z_2}\left[1-\left(\upsilon_G(p_i)^2+\nu_G(p_i)^2+\phi_G(p_i)^2\right)\right] \quad (2.18)$$

Similarly, we get

$$M_2(G\cap H)$$

$$=\frac{1}{n}\sum_{Z_1}\left[1-\left(\upsilon_G(p_i)^2+\nu_G(p_i)^2+\phi_G(p_i)^2\right)\right]$$

$$+\frac{1}{n}\sum_{Z_2}\left[1-\left(\upsilon_H(p_i)^2+\nu_H(p_i)^2+\phi_H(p_i)^2\right)\right] \quad (2.19)$$

Now, adding (2.18)and (2.19), we have

$$M_2(G\cup H)+M_2(G\cap H)=M_2(G)+M_2(H) \quad (2.20)$$

This proves the theorem.

2.3 Picture Fuzzy MADM Based on TODIM Method

Based upon proposed entropy we applied a PF TODIM approach to solve the MADM problems .We choose the best alternative from the set of alternatives which we determined on the basis of collection of data based on some criteria. To show validity and practical reasonability, we apply proposed measure in a MADM problem involving partially known information for criteria weights for alternatives in picture fuzzy information.

TODIM Method

The essential steps for the new Picture fuzzy TODIM approach are briefly as follows: Consider $B=\{\phi_1,\phi_2,\ldots,\phi_m\}$ and $G'=\{\theta_1,\theta_2,\ldots,\theta_m\}$ is a set of alternatives and criterion, respectively. Let $D=d_{ij}$ be a picture fuzzy decision matrix, where d_{ij} denotes the preference information of the alternatives ϕ_i w.r.t. the attribute θ_j is expressed in terms of PFN $d_{ij}=(\upsilon_{ij},\mu_{ij},\eta_{ij}); 1\le i\le m, 1\le j\le n.$ The MADM problem with PFNs depicted in picture fuzzy matrix as:

$$D=[d_{ij}]_{m\times n}=$$

$$\begin{bmatrix} & \theta_1 & \theta_2 & \cdots & \theta_m \\ \phi_1 & (\upsilon_{\beta_{11}},\mu_{\beta_{11}},\eta_{\beta_{11}}) & (\upsilon_{\beta_{12}},\mu_{\beta_{12}},\eta_{\beta_{12}}) & \cdots & (\upsilon_{\beta_{1n}},\mu_{\beta_{1n}},\eta_{\beta_{1n}}) \\ \phi_2 & (\upsilon_{\beta_{21}},\mu_{\beta_{21}},\eta_{\beta_{21}}) & (\upsilon_{\beta_{22}},\mu_{\beta_{22}},\eta_{\beta_{22}}) & \cdots & (\upsilon_{\beta_{2n}},\mu_{\beta_{2n}},\eta_{\beta_{2n}}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_m & (\upsilon_{\beta_{m1}},\mu_{\beta_{m1}},\eta_{\beta_{m1}}) & (\upsilon_{\beta_{m2}},\mu_{\beta_{m2}},\eta_{\beta_{m2}}) & \cdots & (\upsilon_{\beta_{mn}},\mu_{\beta_{mn}},\eta_{\beta_{mn}}) \end{bmatrix} \quad (2.21)$$

Step 1: Transform $D=(d_{ij})_{m\times n}$ into a normalized Picture fuzzy (NPF) decision matrix as follows:

$$q_{ij}=\begin{cases}\text{neg}(d_{ij})^c, & \text{for cost attribute}\\ d_{ij}, & \text{for benefit criteria}\end{cases} \quad (2.22)$$

where $\text{neg}(d_{ij})=(\eta_{ij},\mu_{ij},\upsilon_{ij})$ shows the complement of d_{ij} .Then, we obtain a new PF decision matrix $D^*=(q_{ij})_{m\times n}$

Step 3:

Determination of Completely known criteria weights

We will use the below said method to find the criteria weights:

$$Min \quad \hat{T}=\sum_{j=1}^{n}\Omega_j\sum_{i=1}^{m}M_2(\upsilon_{ij})$$

$$=\frac{1}{n}\sum_{i=1}^{m}\sum_{i=1}^{n}\Omega_j\times\left[1-\left(\upsilon_G(p_i)^2+\nu_G(p_i)^2+\eta_G(p_i)^2+\phi_G(p_i)^2\right)\right]. \quad (2.23)$$

such that $\Omega_j\ge 0, j=1,2,\ldots,n$; $\sum_{j=1}^{n}\Omega_j=1$.

Also

$$\Omega_{jr}=\frac{\Omega_j}{\Omega_r}, j,r=1,2,\ldots,n \quad (2.24)$$

where (Ω_{jr}) is the relative weight of each criteria from the weight of criteria.

where Ω_j is the weight of the attribute $\theta_j,\Omega_r=\max\{\Omega_1,\Omega_2,\ldots,\Omega_n\}$ and $0\le\Omega_{jr}\le 1.$

Step 4: Using (2.24), evaluate the dominance degree of ϕ_i over each alternative ϕ_j concerning each attribute θ_j by the following mathematical model:

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$$Z_j(\Phi_i,\Phi_{t_1}) = \begin{cases} \sqrt{\dfrac{\Omega_{jr} d_H(q_{ij},q_{t_1 j})}{\sum_{j=1}^{n}\Omega_{jr}}}, & \text{if} \quad q_{ij}-q_{t_1 j}>0 \\ 0, & \text{if} \quad q_{ij}-q_{t_1 j}=0 \\ -\dfrac{1}{\gamma}\sqrt{\dfrac{\left(\sum_{j=1}^{n}\Omega_{jr}\right) d_H(q_{ij},q_{t_1 j})}{\Omega_{jr}}} & \text{if} \quad q_{ij}-q_{t_1 j}<0 \end{cases}$$

(2.25)

where $d_H(q_{ij},q_{t_1 j})$ determines the distance between the two PFNs. q_{ij} and $q_{t_1 j}$ and $\gamma > 0$ denotes the attenuation factor of the losses. By definition if $q_{ij} > q_{t_1 j}$, then $Z_j(\Phi_i,\Phi_{t_1})$ signifies a gain ; if $q_{ij} < q_{t_1 j}$, then $Z_j(\Phi_i,\Phi_{t_1})$ signifies losses.

Step 5: Then find the matrix of dominance degree for each alternative Φ_i , w.r.t. the each criteria Θ_j :

$$Z_j = [Z_j(r_i,r_{t_1})]_{m\times m} = \begin{bmatrix} & \Theta_1 & \Theta_2 & \cdots & \Theta_m \\ \Phi_1 & 0 & Z_j(\Phi_1,\Phi_2) & \cdots & Z_j(\Phi_1,\Phi_m) \\ \Phi_2 & Z_j(\Phi_2,\Phi_1) & 0 & & Z_j(\Phi_2,\Phi_m) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Phi_m & Z_j(\Phi_m,\Phi_1) & Z_j(\Phi_m,\Phi_2) & \cdots & 0 \end{bmatrix}$$

(2.26)

Step 6:Find the totally dominance degree of each alternative Φ_i w.r.t. another alternatives $\Phi_{t_1}(t_1 = 1,2,\dots,m)$ by using:

$$\Delta_j(\Phi_i,\Phi_{t_1}) = \sum_{t_1=1}^{m} Z_j(\Phi_i,\Phi_{t_1})$$

(2.27)

Hence , by using equation (2.27), the overall dominance degree matrix can be obtained as :

$$Z = [Z_j(\Phi_i,\Phi_{t_1})]_{m\times m} = \begin{bmatrix} & \Theta_1 & \Theta_2 & \cdots & \Theta_m \\ \Phi_1 & 0 & Z_j(\Phi_1,\Phi_2) & \cdots & Z_j(\Phi_1,\Phi_m) \\ \Phi_2 & Z_j(\Phi_2,\Phi_1) & 0 & & Z_j(\Phi_2,\Phi_m) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Phi_m & Z_j(\Phi_m,\Phi_1) & Z_j(\Phi_m,\Phi_2) & \cdots & 0 \end{bmatrix}$$

(2.28)

Step 7: Finally, overall dominance degree of each alternative Φ_i will be obtained by using the below said formula:

$$\xi(\Phi_i) = \frac{\sum_{t_1=1}^{m} Z_j(\Phi_i,\Phi_{t_1}) - min_i\left(\sum_{t_1=1}^{m} Z_j(\Phi_i,\Phi_{t_1})\right)}{max_i\left(\sum_{t_1=1}^{m} Z_j(\Phi_i,\Phi_{t_1})\right) - min_i\left(\sum_{t_1=1}^{m} Z_j(\Phi_i,\Phi_{t_1})\right)}$$

(2.29)

where $0 \le \xi(\Phi_i) \le 1$. On the value of $\xi(\Phi_i)$, the rank of each alternative Φ_i is dependent. More the value of $\xi(\Phi_i)$ is, the better the alternative Φ_i is. Finally, the measures computed by (2.29) permit the order ranking of all alternatives.

3 Results

3.1 The Application of Picture Fuzzy TODIM Approach

Muthoot Finance Limited (MFL) is the most profitable and broadened non-bank in India with a wide arrangement of items spread across consumer, SME (Small and Medium-sized enterprises) and wealth management as well as commerical lending. (MFL) which was formed in the area of financial services for the buisness. It serves a large number of consumers in the financial services space by providing solutions for resource obtaining through general insurance, financing, income and family protection in the form of medical and life insurance. Based on some preparatory work, MFL makes further efforts in many areas. A problem, which has obtain a lot of attention in the whole country, is the choice of the governor of MFL, which considers a MADM problem. For this purpose, we put this problem in the MADM environment and simulate a decision making procedure of this choice problem. The choice of governor of MFL depends on some criteria, like management skill (Φ_1), the ability of communicating (Φ_2), the ability of establishing stable financial mechanisms (Φ_3), the degree of familiarity of governmental issues (Φ_4), basic education (Φ_5).

Decision analysis with proposed method in PF environment analysis

Step 1. Construct a (PF) decision matrix as(All criteria are taken to be benefit criteria):

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Table 1: PF-decision matrix

Decision value	θ_1	θ_2	θ_3	θ_4	θ_5
Φ_1	(0.1,0.2,0.4)	(0.6,0.1,0.1)	(0.1,0.2,0.6)	(0.4,0.1,0.4)	(0.1,0.4,0.2)
Φ_2	(0.6,0.1,0.2)	(0.4,0.3,0.1)	(0.5,0.1,0.3)	(0.2,0.3,0.4)	(0.2,0.3,0.4)
Φ_3	(0.6,0.1,0.3)	(0.2,0.4,0.2)	(0.8,0.0,0.1)	(0.2,0.4,0.1)	(0.4,0.4,0.1)
Φ_4	(0.1,0.3,0.5)	(0.5,0.2,0.2)	(0.2,0.3,0.2)	(0.6,0.1,0.2)	(0.5,0.2,0.1)
Φ_5	(0.3,0.3,0.2)	(0.2,0.6,0.1)	(0.4,0.2,0.3)	(0.1,0.1,0.6)	(0.6,0.1,0.3)

Step 2. The attribute weight with partial information are given as :

$$\Omega = \{0.12 \le \Omega_1 \le 0.27, 0.16 \le \Omega_2 \le 0.19, \quad 0.28 \le \Omega_3 \le 0.39, 0.19 \le \Omega_4 \le 0.42, 0.15 \le \Omega_5 \le 0.30\}.$$

The overall entropy of each attribute determined as foolows :

$K_1 = \sum_{i=1}^{5} \upsilon_{1j} = \sum_{i=1}^{5} V_\omega(q_{1j}) = 0.9857; K_2 = \sum_{i=1}^{5} \upsilon_{2j} = \sum_{i=1}^{5} V_\omega(q_{2j}) = 0.8656;$

$K_3 = \sum_{i=1}^{5} \upsilon_{3j} = \sum_{i=1}^{5} V_\omega(q_{3j}) = 0.9253; K_4 = \sum_{i=1}^{5} \upsilon_{4j} = \sum_{i=1}^{5} V_\omega(q_{4j}) = 0.6086;$

$K_5 = \sum_{i=1}^{5} \upsilon_{5j} = \sum_{i=1}^{5} V_\omega(q_{5j}) = 0.9277.$

We used the following optimization technique to find the weights ;

Min

$\Omega = 0.9857\Omega_1 + 0..8657\Omega_2 + 0.9253\Omega_3 + 0.6086\Omega_4 + 0.9277\Omega_5$

$\text{suchthat} \sum_{j=1}^{5} \Omega_j = 1 \ and \ \Omega_j \ge 0, j = 1,2,3,4,5.$.

The weighting vector obtained as: $\Omega = (0.22,0.16,0.28,0.19,0.15)^T$.

Step 3. By assuming $\gamma = 2.5$, we construct the dominance matrix $Z_1 - Z_5$ as follows:

$$Z_1 = \begin{bmatrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 \\ \Phi_1 & 0.0000 & 0.2422 & 0.2366 & -0.2202 & 0.1915 \\ \Phi_2 & -0.4404 & 0.0000 & -0.1557 & -0.3924 & -0.2481 \\ \Phi_3 & -0.4119 & 0.0856 & 0.0000 & -0.4404 & -0.3813 \\ \Phi_4 & 0.1211 & 0.2708 & 0.2422 & 0.0000 & 0.1915 \\ \Phi_5 & -0.2481 & 0.1915 & 0.2098 & -0.2481 & 0.0000 \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 \\ \Phi_1 & 0.0000 & -0.4272 & -0.5164 & -0.4762 & -0.5477 \\ \Phi_2 & 0.3161 & 0.0000 & -0.2651 & 0.0000 & -0.1782 \\ \Phi_3 & 0.2066 & 0.1461 & 0.0000 & 0.1633 & 0.1633 \\ \Phi_4 & 0.1265 & 0.0000 & -0.2982 & 0.0000 & -0.5164 \\ \Phi_5 & 0.2191 & 0.1633 & -0.1782 & 0.2066 & 0.0000 \end{bmatrix}$$

$$Z_3 = \begin{bmatrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 \\ \Phi_1 & 0.0000 & 0.2132 & 0.2778 & 0.2166 & 0.2166 \\ \Phi_2 & -0.2903 & 0.0000 & 0.2166 & -0.3381 & -0.1952 \\ \Phi_3 & -0.5164 & -0.3381 & 0.0000 & -0.4364 & -0.2903 \\ \Phi_4 & -0.3381 & 0.2366 & 0.3055 & 0.0000 & 0.1932 \\ \Phi_5 & -0.3381 & 0.1166 & 0.2132 & -0.4760 & 0.0000 \end{bmatrix}$$

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$Z_4 =$

$$\begin{bmatrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 \\ \phi_1 & 0.0000 & -0.3751 & 0.2251 & 0.2592 & -0.3746 \\ \phi_2 & 0.2592 & 0.0000 & 0.2599 & 0.2251 & -0.3746 \\ \phi_3 & -0.4739 & -0.3351 & 0.0000 & 0.2251 & -0.5026 \\ \phi_4 & -0.3751 & -0.4739 & -0.4739 & 0.0000 & -0.5026 \\ \phi_5 & 0.1679 & 0.1679 & 0.1387 & 0.1387 & 0.0000 \end{bmatrix}$$

$Z_5 =$

$$\begin{bmatrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 \\ \phi_1 & 0.0000 & -0.4971 & 0.1414 & 0.2871 & 0.1121 \\ \phi_2 & 0.1414 & 0.0000 & 0.4732 & 0.2871 & 0.2871 \\ \phi_3 & -0.4971 & -0.4619 & 0.0000 & 0.1225 & 0.0000 \\ \phi_4 & -0.4989 & -0.4989 & -0.3266 & 0.0000 & -0.4971 \\ \phi_5 & -0.5657 & -0.4989 & 0.0000 & 0.1414 & 0.0000 \end{bmatrix}$$

Step 4. Totally dominance degree obtained as :

D=

$$\begin{bmatrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 \\ \phi_1 & 0.0000 & -0.8440 & 0.3645 & 0.0665 & -0.4021 \\ \phi_2 & -0.0140 & 0.0000 & 0.5289 & -0.2183 & -0.7090 \\ \phi_3 & -1.6927 & -0.9034 & 0.0000 & -0.3659 & -1.0109 \\ \phi_4 & -0.9645 & -0.4654 & -0.5510 & 0.0000 & -1.1314 \\ \phi_5 & -0.7649 & 0.1404 & 0.3835 & -0.2374 & 0.0000 \end{bmatrix}$$

Step 5. Determine the overall dominance degree $\xi(\phi_i)$ of alternatives ϕ_i using equation (2.29)

$$\xi(\phi_1) = 0.8869, \xi(\phi_2) = 1.000, \xi(\phi_3) = 0.0000, \xi(\phi_4) = 0.2417 \xi(\phi_5) = 0.9815$$

Step 6. Ranking of the alternatives is $\phi_2 > \phi_5 > \phi_1 > \phi_4 > \phi_3$ and the result shows that ϕ_4 is the most optimal choice of the governor of MFL.

3.2 Comparative Analysis

The above example was solved by using the existing methods proposed by [4, 15, 18, 27] with same attribute weights information and results are depicted in Table 2.

Table 2: Ranking results

Methods Proposed by	Ranking method	Ranking
Wei,2016 [4]	Cross entropy	$\phi_2 \succ \phi_4 \succ \phi_5 = \phi_3 \succ \phi_1$
Amalendu et al.,2019[18]	New ranking method	$\phi_2 \succ \phi_5 \succ \phi_4 \succ \phi_3 \succ \phi_1$
Chunxin Bo et al,2017[27]	Score function	$\phi_2 \succ \phi_3 \succ \phi_5 \succ \phi_1 \succ \phi_4$
Nei , 2018 [15]	Comparison rule	$\phi_2 = \phi_3 \succ \phi_5 \succ \phi_1 \succ \phi_4$
New method	TODIM	$\phi_2 > \phi_5 > \phi_1 > \phi_4 > \phi_3$

The ranking of alternatives so obtained is given by : $\phi_2 > \phi_5 > \phi_1 > \phi_4 > \phi_3$ with ϕ_2 as the most suitable alternative. In our proposed method ϕ_2 is best choice but ranking order does not matter for other alternatives.

In subsequent works, more methods with PFNs should be done to measure the uncertainity in decision making and to analyse the risk [14,23-25,30, 31, 32,]. Hence, the results of proposed approach are more reasonable and simple.

4 Conclusions

PFSs are suitable in managing and tending the uncertainty and vagueness information measure that occurred in MADM problems. A new entropy measure under picture fuzzy environment has effectively built up. The entropy evaluation model is used to determine criteria weights. Then, a model of MADM is presented to compute the information of the PFSs. Moreover, the viability of the proposed technique is throughly clarified with the help of a numerical model by using TODIM method. To check the viability and objectivity of the proposed MADM approach , we compare the resulting output with the existing techniques to solve theMADM problems . The proposed MADM approach can can likewise be utilized to tackle the convoluted issues like insurance sector,finance sector , site choice etc.

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