Integral transforms of (p,s,k) Mittag-Leffler function ${}_pE^{\rho,s}_{k,\theta,\vartheta}(z)$

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Abstract

In this paper, we evaluate Melin-Barnes integral representation of (p,s,k) Mittag Leffler function. Sumudu transform have been used in the generaliztion of Mittag Leffler function Also we establish some corollary of a special cases. Some new result are our finding.

MSC(2010):

Keywords: (p,s,k) Mittag-Leffler Function,Mellin transform, Summudu transform.

1 INTRODUCTION AND PRELIMINARIES

In this paper study with Mittag-Leffler function and its generliazations. Its importance is realized during the last one and half decades due to its direct involvement in the problem of physics, biology, engineering and applied sciences. Mittag-Leffler function naturally occurs as the solution of fractional order differential equation and fractional order integral equations.

1.1

Mittag-Leffler function
Suppose that $\theta \in \mathbb{C}, Re(\theta) > 0, z \in \mathbb{Z}$ then Mittag-Leffler function defineed by

$$E_\theta(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(1+n\theta)} \tag{1}$$

$$E_{\theta,\vartheta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\vartheta+n\theta)} \tag{2}$$

$\theta, \vartheta \in \mathbb{C}, Re(\theta) > 0, Re(\vartheta) > 0 z \in \mathbb{Z}$ Recently in 2012, G.A. Dorrego, and R.A. cerutti introduce

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the K-Mittag Leffler function $E^{\varrho}_{k,\theta,\vartheta}(z)$ defined by

$$E^{\varrho,s}_{k,\theta,\vartheta}(z) = \frac{(\varrho)_{n,k} z^n}{\Gamma_k(n\theta + \vartheta)n!} \tag{3}$$

where k$\in \mathbb{R}, \theta, \vartheta, \rho \in \mathbb{C} Re(\theta) > 0, Re(\vartheta) > 0,$
Let $k, p \in R^+ - \{0\}; \theta, \vartheta, \rho \in C/kZ^-; Re(\theta) > 0, Re(\vartheta) > 0, Re(\rho) > 0$ and $s \in (0,1) \cup N$.
The p - k Mittag-Leffler function denoted by ${}_pE^{\rho,s}_{k,\theta,\vartheta}(z)$ and defined as

$${}_pE^{\rho,s}_{k,\theta,}(z) = \sum_{n=0}^{\infty} \frac{{}_p(\rho)_{ns,k}}{{}_p\Gamma_k(n\theta + \vartheta)} \frac{z^n}{n!}. \tag{4}$$

Suppose that $k, p, s \in \mathbb{R}, \theta, \vartheta, \rho \in \mathbb{C} Re(\theta) > 0, Re(\vartheta) > 0 and Re(\rho) > 0$ then Mittag-Leffer (p, s, k) function defined by

$${}_pE^{\varrho,s}_{k,\theta,\vartheta}(z) = \sum_{n=0}^{\infty} \frac{{}_p(\varrho)_{n,k,s} z^n}{{}_p\Gamma_{1,k}(n\theta + \vartheta)n!}. \tag{5}$$

1.2 Special cases of Mittag-Leffler function

(1) For s = 1 equation (2.7) reduces in the Mittag-Leffler(p-k)-function defined in [17]

$${}_pE^{\varrho,1}_{k,\theta,\vartheta}(z) = \sum_{n=0}^{\infty} \frac{{}_p(\varrho)_{n,k,1} z^n}{{}_p\Gamma_{1,k}(n\theta + \vartheta)n!}. \tag{6}$$

(2) For p=1 ans s=1, equation (2.7) reduce in the Mittag-Leffler k-function defined in [3]

$${}_1E^{\varrho,1}_{k,\theta,\vartheta}(z) = \sum_{n=0}^{\infty} \frac{{}_1(\varrho)_{n,k,1} z^n}{{}_1\Gamma_{1,k}(n\theta + \vartheta)n!}. \tag{7}$$

(3) For k=1, p=1 and s=1 equation (2.7) reduce in the Mittag-Leffler defined in[12]

$${}_1E^{\varrho,1}_{1,\theta,\vartheta}(z) = \sum_{n=0}^{\infty} \frac{{}_1(\varrho)_{n,1,1} z^n}{{}_1\Gamma_{1,1}(n\theta + \vartheta)n!}. \tag{8}$$

(4) For k=1, p=1, s=1 and $\varrho = 1$, equation (2.7) reduce in the Mittag-Leffler defined in [2]

$${}_1E^{\varrho,1}_{1,\theta,\vartheta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\theta + \vartheta)n!}. \tag{9}$$

(5) For k=1, p=1, s=1 $\vartheta = 1$and $\varrho = 1$, equation (2.7) reduce in the Mittag-Leffler k-function defined in [4]

$${}_1E^{1,1}_{1,\theta,1}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\theta + 1)n!}. \tag{10}$$

(6) If k=1, p=1, s=1 $\theta = 1 \vartheta = 1 \varrho = 1$, then we get the exponential function.

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1.3 Mellin Transform

$$M[f(z):s] = \int_0^\infty z^{s-1} f(z)dz = f^*(s), Re(s) > 0, \tag{11}$$

then

$$f(z) = M^{-1}[f^*(s):x] = \int f^*(s)x^{-s}ds, \tag{12}$$

1.4 Sumudu Transform

The integral transform very efficient to solve differential equations well asused in science astronomy and physics.Watugala [35] introduce a new transform and named Summudu transform has defined by the following formula:

$$\mathcal{A} = f(t)|\exists \mathfrak{M}, \eta_1, \eta_2 > 0, |f(t)| < \mathfrak{M}e^{\frac{|t|}{r_j}}, \ t \in (-1)^j \times [0, \infty).$$

$\mathsf{G}(\tau) = S[f(t);\tau] = \int_0^\infty e^- t f(\tau t)dt; \tau \in (-\eta_1, -\eta_2)$

Theorem 1.1 : The relation between three parameters,two parameters, and the classical Pochhamer's symbol is given by

$${}_p[\xi]_{n,k,s} = s_p^n[\xi]_{n,k} = (\frac{sp}{k})^n(\xi)_{n,k} = (sp)^n(\frac{\xi}{k})_n.$$

Proof: the proof of theorem see[2]

Theorem 1.2 The relation between gamma (p,s,k) function . gamma k-function and classic gamma function is given by

${}_p\Gamma_{k,s}(\xi) = (s)^\xi/k_p\Gamma_k(\xi) = (\frac{sp}{k})^\xi/k_p\Gamma_k(\xi) = \frac{(sp)^\xi/k}{k}\Gamma(\frac{\xi}{k})$

Proof: the proof of theorem see[2]

Main Result

1.5 Mellin- Barnes integral reprentation of (p,s,k) Mittag-Leffler function ${}_pE^{\varrho,s}_{k,\theta,\vartheta}(z)$

In this section , we will present the techniques of Mellin Barnes integral it has been used in the particle physics because of their importance in the inversionof the Laplace and Mellin transforms. Therefore we will evaluate of some contour integral with the help of residuce theorem with involving (p,s,k) Mittag-Leffler function.

Theorem 2.1 Let k,p ,q > and $\theta, \vartheta, \varrho$ with Re(θ) $> 0, Re(\vartheta) > 0, Re(\varrho) > 0$, then (p,s,k) Mittag-Leffler function ${}_pE^{\varrho,s}_{k,\theta,\vartheta}(z)$ is represented by the Mellin-Barnes integral as

$${}_pE^{\varrho,q}_{k,\theta,\vartheta}(z) = \frac{k(qp)^{-\frac{\vartheta}{k}}}{2\pi i\Gamma(\frac{\varrho}{k})} \int_L \frac{\Gamma(s)\Gamma((\frac{\varrho}{k}) - s)}{\Gamma((\frac{\vartheta}{k}) - (\frac{\theta s}{k})}(-z(qp)^{1-\frac{\theta}{k}})^{-s}ds. \tag{13}$$

Where $| \, argz \, | < \pi$; the contour integration beginning at $-i\infty$ and ending at $+i\infty$, and indented to separate the poles of the integrand as $s = -n foreveryn \in N_0$(to the left) from those at $s = \frac{\varrho}{k} + n foreveryn \in N_0$(to the right).

Proof Consider the integral on right side of equation(2.1) and use the theorem of calculus of

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residues,

$$A \equiv \frac{k(qp)^{-\frac{\vartheta}{k}}}{2\pi i \Gamma(\frac{\varrho}{k})} \int_L \frac{\Gamma(s)\Gamma((\frac{\varrho}{k}) - s)}{\Gamma((\frac{\vartheta}{k}) - (\frac{\theta s}{k})}(-z(qp)^{1-\frac{\theta}{k}})^{-s} ds$$

$= 2\pi i$[sum of the residues at the poles $s = 0, -1, -2, ...$]

$$A \equiv \frac{k(qp)^{-\frac{\vartheta}{k}}}{\Gamma(\frac{\varrho}{k})} \sum_{n=0}^{\infty} \underset{s=-n}{Re} \quad s[\frac{\Gamma(s)\Gamma((\frac{}{k}) - s)}{\Gamma((\frac{\vartheta}{k}) - (\frac{\theta s}{k})}](-z(qp)^{1-\frac{\theta}{k}})^{-s}$$

$$A \equiv \frac{k(qp)^{-\frac{\vartheta}{k}}}{\Gamma(\frac{\varrho}{k})} \sum_{n=0}^{\infty} \lim_{s \to -n} \frac{\pi(s+n)}{sin \pi s} \frac{\Gamma((\frac{\varrho}{k}) - s)}{\Gamma(1-s)\Gamma((\frac{\vartheta}{k}) - (\frac{\theta s}{k})}](-z(qp)^{1-\frac{theta}{k}})^{-s}$$

$$k(qp)^{-\frac{\vartheta}{k}} \sum_{n=0}^{\infty} \frac{(\frac{\varrho}{k})_n}{\Gamma((\frac{\vartheta}{k}) + (\frac{\theta n}{k})} \frac{(z(qp)^{1-\frac{\theta}{k}})^n}{n!}$$

using equations (1.14) and (1.17), we have,

$$A \equiv_p E_{k,\theta,\vartheta}^{\varrho,q}(z)$$

Hence.

1.6 Sumudu Transform of (p,s,k) Mittag-Leffler function ${}_pE_{k,\theta,\vartheta}^{\varrho,s}(z)$

In this work our aim is to exhibit solution of a (p,s,k) Mittag-Leffler function by using Sumudu transform.

Theorem 2.2:Let k,p,s,c > 0 and $\theta, \vartheta, \rho \in \mathbb{C} with Re(\theta) > 0, Re(\vartheta) > 0$ and $Re(\rho) > 0$. Then , the Sumudu transform form of ${}_pE_{k,\theta,\vartheta}^{\varrho,s}$ is given by

$$\mathcal{S}(z_p^{v/k-1} E_{k,\theta,\vartheta}^{\varrho,s}(cz)^{\theta/k}) = \frac{k}{(sp)^{v/k}}(1 + ps(\frac{c}{sp})^{\theta/k})^{-\rho/k} \tag{14}$$

$\mathcal{S}(z_p^{v/k-1} E_{k,\theta,\vartheta}^{\varrho,s} \pm (cz)^{\theta/k}) = \int_0^\infty e^{-z} z^{v/k-1} \sum_{n=0}^\infty \frac{{}_p(\varrho)_{n,k,s}}{{}_p\Gamma_{s,k}(n\theta + \vartheta)} (\pm 1)^n (cz)^{n\theta/k}$

$= \sum_{n=0}^\infty \frac{{}_p(\varrho)_{n,k,s}}{{}_p\Gamma_{s,k}(n\theta + \vartheta)} \frac{c^{n\theta/k}}{n!} \int_0^\infty e^{-z} z^{\theta/k+\vartheta/k-1} dz$

$= \sum_{n=0}^\infty \frac{{}_p(\varrho)_{n,k,s}}{{}_p\Gamma_{s,k}(n\theta + \vartheta)} \frac{c^{n\theta/k}}{n!} \int_0^\infty e^{-z} z^{n\theta/k+\vartheta/k-1} dz$

$= \sum_{n=0}^\infty \frac{{}_p(\varrho)_{n,k,s}}{{}_p\Gamma_{s,k}(n\theta + \vartheta)} \frac{c^{n\theta/k}}{n!} \Gamma(\frac{\theta n + \varrho}{k})$

Use theorem 1.1 and 1.2.

$= \sum_{n=0}^\infty \frac{(\varrho)_n, s}{n!} [(\frac{c}{sp})^\theta / k \frac{sp}{k}]^n.$

$= \frac{k}{(sp)^{\vartheta/k}} (1 \mp ps(\frac{c}{sp})^{\theta/k})) - \varrho/k.$

1.7 Mellin Transform of (p,s,k) Mittag-Leffler function ${}_pE_{k,\theta,\vartheta}^{\varrho,s}(z)$

Theorem 2.3 The Mellin transform of (p,s,k) Mittag-Leffler function, ${}_pE_{k,\theta,\vartheta}^{\varrho,s}(z)$,

$$\int_0^\infty t^{s-1} {}_pE_{k,\theta,\vartheta}^{\varrho,q}(-wt) ds = \frac{\Gamma(s), \Gamma(\frac{\varrho}{k} - s)}{\Gamma(\frac{\varrho}{k})\Gamma((\frac{\vartheta}{k} - (\frac{\theta s}{k})} (\frac{-(qp)^{\frac{\vartheta}{k}-1}}{w})^s ds \tag{15}$$

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Where $k, p \in R^{+} - \{0\}$; $a, \sigma \in C$; $Re(\alpha) > 0, Re(\beta) > 0, Re(\gamma) > 0, Re(s) > 0$ and $q \in (0,1) \cup N$.
Proof Putting z = -wt in equation (11), we have

$$ {}_pE^{\varrho,q}_{k,,\vartheta}(-wt) = \frac{k(qp)^{-\frac{\vartheta}{k}}}{2\pi i \Gamma(\frac{\varrho}{k})} \int_L \frac{\Gamma(s)\,\Gamma(\frac{\varrho}{k} - s)}{\Gamma(\frac{\vartheta}{k})\Gamma(\frac{\theta}{k} - \frac{\theta s}{k})} (-wt(qp)^{1-\frac{}{k}})^{-s} ds $$

$$ {}_pE^{\varrho,q}_{k,\theta,\vartheta}(-wt) = \frac{k(qp)^{-\frac{\vartheta}{k}}}{2\pi i \Gamma(\frac{\varrho}{k})} \int_L f^*(s)(t)^{-s} ds. \tag{16} $$

where

$$ f^*(s) = \frac{\Gamma(s), \Gamma(\frac{\varrho}{k} - s)}{\Gamma(\frac{\vartheta}{k} - \frac{\theta s}{k})} (-wt(qp)^{1-\frac{\vartheta}{k}})^{-s} $$

1.8 Special Cases

Corolary 3.1 Let k,p> and $\theta, \vartheta, \rho \in \mathbb{C} with Re(\theta) > 0, Re(\vartheta) > 0, Re(\varrho) > 0$, then the(p,k) Mittag-Leffler function ${}_pE^{\varrho,q}_{k,\theta,\vartheta}(z)$ is represented by the Mellin-Barnes integral as

$$ {}_pE^{\varrho,1}_{k,\theta,\vartheta}(z) = \frac{k(p)^{-\frac{\vartheta}{k}}}{2\pi i \Gamma(\frac{\varrho}{k})} \int_L \frac{\Gamma(s)\Gamma((\frac{\varrho}{k}) - s)}{\Gamma((\frac{\vartheta}{k}) - (\frac{\theta s}{k})} (-z(qp)^{1-\frac{\theta}{k}})^{-s} ds. \tag{17} $$

Where $\mid argz \mid < \pi$; the contour integration beginning at $-i\infty$ and ending at $+i\infty$, and indented to separate the poles of the integrand as $s = -n for every n \in N_0$(to the left) from those at $s = \frac{\varrho}{k} + n for every n \in N_0$(to the right).
Proof :The proof of the corollary are similar with the proof of theorem(2.2) put q = 1 .

Corolary 3.2 Let k,p> and $\theta, \vartheta, \rho \in \mathbb{C} with Re(\theta) > 0, Re(\vartheta) > 0, Re(\varrho) > 0, and$ then the function ${}_pE^{\varrho,s}_{k,\theta,\vartheta}(z)$ is represented by the Mellin-Barnes integral as

$$ {}_1E^{\varrho,1}_{k,\theta,\vartheta}(z) = \frac{k}{2\pi i \Gamma(\frac{\varrho}{k})} \int_L \frac{\Gamma(s)\Gamma((\frac{\varrho}{k}) - s)}{\Gamma((\frac{\vartheta}{k}) - (\frac{\theta s}{k})} (-z^{-s} ds. \tag{18} $$

Where $\mid argz \mid < \pi$; the contour integration beginning at $-i\infty$ and ending at $+i\infty$, and indented to separate the poles of the integrand as $s = -n for every n \in N_0$(to the left) from those at $s = \frac{\varrho}{k} + n for every n \in N_0$(to the right).
Proof The proof of the corollary are similar with the proof of theorem(2.2) put q = 1 and p = 1.
Corolary 3.3 Let $Re(\theta) > 0, Re(\vartheta) > 0, Re(\varrho) > 0$, then the function ${}_1E^{\varrho,1}_{1,\theta,\vartheta}(z)$ is represented by the Mellin-Barnes integral as

$$ {}_1E^{\varrho,1}_{1,\theta,\vartheta}(z) = \frac{1}{2\pi i \Gamma(\varrho)} \int_L \frac{\Gamma(s)\Gamma((\varrho) - s)}{\Gamma \vartheta - \theta s})(-z^{-s} ds. \tag{19} $$

Where $\mid argz \mid < \pi$; the contour integration beginning at $-i\infty$ and ending at $+i\infty$, and indented to separate the poles of the integrand as $s = -n for every n \in N_0$(to the left) from those at $s = \varrho + n for every n \in N_0$(to the right).
ProofThe proof of the corollary are similar with the proof of theorem(2.2) put q = 1,k = 1 and p = 1.
Corolary 3.4 Let $Re(\theta) > 0, Re(\vartheta) > 0$, then the function ${}_1E^{1,1}_{1,\theta,\vartheta}(z)$ is represented by the Mellin-Barnes integral as

$$ {}_1E^{1,1}_{1,\theta,\vartheta}(z) = \frac{1}{2\pi i \Gamma(\varrho)} \int_L \frac{\Gamma(s)\Gamma((\varrho) - s)}{\Gamma \vartheta - \theta s})(-z^{-s} ds. \tag{20} $$

Where $\mid argz \mid < \pi$; the contour integration beginning at $-i\infty$ and ending at $+i\infty$, and indented to separate the poles of the integrand as $s = -n for every n \in N_0$(to the left) from those at

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$s = nforeveryn \in N_0$(to the right).
ProofThe proof of the corollary are similar with the proof of theorem(2.2) put q = 1,k = 1 ,p = 1 and $\varrho = 1$.
Corolary 3.5 Let $Re(\theta) > 0$ then the function ${}_1E^{1,1}_{1,\theta,\vartheta}(z)$ is represented by the Mellin-Barnes integral as

$$ {}_1E^{1,1}_{1,\theta,1}(z) = \frac{1}{2\pi i}\int_L \frac{\Gamma(s)\Gamma(1-s)}{\Gamma(1-\theta s)}(-z^{-s}ds. \tag{21} $$

Where $\mid argz \mid < \pi$; the contour integration beginning at $-i\infty$ and ending at $+i\infty$, and indented to separate the poles of the integrand as $s = -nforeveryn \in N_0$(to the left) from those at $s = nforeveryn \in N_0$(to the right).
ProofThe proof of the corollary are similar with the proof of theorem(2.2) put q = 1,k = 1 ,p = 1 $\vartheta = 1$ and $\varrho = 1$.
Corrolary 3.6:Let Let k,p,c > 0 $\theta, \vartheta, \rho \in \mathbb{C}withRe(\theta) > 0, Re(\vartheta) > 0$ and $Re(\rho) > 0$. Then , the Sumudu transform form of p-k Mittag -Leffler function ${}_pE^{\varrho,1}_{k,\theta,\vartheta}$ is given by

$$ \mathcal{S}(z_1^{v/k-1}E^{\varrho,1}_{k,\theta,\vartheta}(\pm(cz)^{\theta/k}) = \frac{k}{(p)^{v/k}}(1 + \mp p(\frac{c}{p})^{\theta}/k)^{-}\rho/k \tag{22} $$

proof:The proof of the corollary are similar with the proof of theorem(2.2) put s=1 .
Corrolary 3.7:Let k,c > 0 and $\theta, \vartheta, \rho \in \mathbb{C}withRe(\theta) > 0, Re(\vartheta) > 0$ and $Re(\rho) > 0$. Then , the Sumudu transform form of Mittag Leffler k -function ${}_1E^{\varrho,1}_{k,\theta,\vartheta}$ is given by

$$ \mathcal{S}(z_1^{v/k-1}E^{\varrho,1}_{k,\theta,\vartheta}(\pm(cz)^{\theta/k})) = k(1 \mp c)^{\theta/k})^{-\rho/k} \tag{23} $$

proof:The proof of the corollary are similar with the proof of theorem(2.2) put s=1 and p=1.
Corrolary 3.8:Let c > 0 and $\theta, \vartheta, \rho \in \mathbb{C}withRe(\theta) > 0, Re(\vartheta) > 0$ and $Re(\rho) > 0$. Then , the Sumudu transform form of Mittag Leffler function ${}_1E^{\varrho,1}_{1,\theta,\vartheta}$ is given by

$$ \mathcal{S}(z_1^{v-1}E^{\varrho,1}_{1,\theta,\vartheta}(\pm(cz)^{\theta})) = (1 \mp c^{\theta})^{-\rho} \tag{24} $$

proof:The proof of the corollary are similar with the proof of theorem(2.2) put s=1,k=1 and p=1.
Corrolary 3.9:Let c > 0 and $\theta, \vartheta, \rho \in \mathbb{C}withRe(\theta) > 0, Re(\vartheta) > 0$ and $Re(\rho) > 0$. Then , the Sumudu transform form of Mittag Leffler function ${}_1E^{1,1}_{1,\theta,\vartheta}$ is given by

$$ \mathcal{S}(z_1^{v-1}E^{1,1}_{1,\theta,\vartheta}(\pm(cz)^{\theta}) = (1 \mp c^{\theta})^{-1} \tag{25} $$

proof:The proof of the corollary are similar with the proof of theorem(2.2) put s=1, k=1 , p=1 and $\varrho = 1$.
Corrolary 3.10:Let c > 0 and $\theta, \rho \in \mathbb{C}withRe(\theta) > 0, and$ Re(ρ) > 0. Then , the Sumudu transform form of Mittag Leffler function ${}_1E^{1,1}_{1,\theta,1}$ is given by

$$ \mathcal{S}(z_1^{v-1}E^{1,1}_{1,\theta,1}(\pm(cz)^{\theta})) = (1 \mp c^{\theta})^{-1} \tag{26} $$

proof:The proof of the corollary are similar with the proof of theorem(2.2) put s=1, k=1 , p=1 $\vartheta = 1and\varrho = 1$.

2 CONCLUSION

In this paper we evaluate Mellin-Barnes integral representation of (p,s,k) Mittag- Leffler function with their several special cases. We obtained its Mellin transform, Summudu transform and several special cases.In this above solution the obtained finding will help in generating new result and further studying with new ideas.

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